15-150

Principles of Functional Programming

Slides for Lecture 1

Introduction, Philosophy, Some Basics

January 14, 2025

15-150

Principles of Functional Programming

Michael Erdmann Dilsun Kaynar

Aileen Guo, Andrew Lam, Jacky Gao, Kiera O'Flynn

Alan Abraham, Alex Xu, Alice Tran, Anna Gu, Isaac Li,
Annie Zhang, Brandon Dong, Caroline Shi, Ivy Li,
Daniel Brown, Daniel Ragazzo, Emma Tong, Eric Xu,
Ethan Huang, Janise Kim, Madison Zhao, Nia Robinson,
Stephen Mao, Ting Chen, Xiao Yuan, Nathan Porter,
Jerry Song, Rong Yuan, Megan Han, Lillian Yu, Amy Ma,
Alison Ding, Eric Feng, Alex Willoughby, Chenyun Yang,
Owen Lalis, Rachel Du, Meera Pradeepan, Annie Wang,
Monica Wan, Andrew Zhou, Elijah Rosen, Xavier Lien

How to Succeed in College

SHOW UP

How to Succeed in College

SHOW UP

Go to lecture, go to lab.

Do not expect to understand everything in real-time.

Repeated exposure is important.

How to Succeed in College

Take Notes

By writing.

The eye-hand-brain loop is magical.

How to Succeed in College

Study Your Notes

Study Your Notes

The same day. And again.

And again. And again.

Figure out what you don't understand.

Repeated exposure is important.

How to Succeed in College

Keep up

Do not let work pile up.

Small steps, big achievements.

How to Succeed in College

SHOW UP

Take Notes

Study Your Notes

Keep up

Course Webpage

http://www.cs.cmu.edu/~15150/

Policies: http://www.cs.cmu.edu/~15150/policy.html

Lectures: http://www.cs.cmu.edu/~15150/lect.html

Course Philosophy

Computation is Functional.

Programming is an explanatory linguistic process.

Computation is Functional

values : types

expressions

Functions map values to values

Imperative	vs.	Functional
Command		Expression
↓		↓
• executed • has an effect		• evaluated • no effect
x := 5		3 + 4
(state)		(value)

Programming as Explanation

Problem statement

↓

high expectation to explain precisely & concisely

- invariants
- specifications
- proofs of correctness
- code

Analyze, Decompose & Fit, Prove

Parallelism

		/\
< 1, 0, 0, 1, 1 >	➔	3,
< 1, 0, 1, 1, 0 >	➔	3,
< 1, 1, 1, 0, 1 >	➔	4,
< 0, 1, 1, 0, 0 >	➔	2,
		\/
		↓
		12

Parallelism

sum : int sequence $\rightarrow$ int

type row = int sequence

type room = row sequence

```
fun count (class : room) : int =
        sum (map sum class)
```

Parallelism

- Work:
  - Sequential Computation
  - Total sequential time;

    number of operations

- Span:
  - Parallel Computation
  - How long would it take if one could have as many processors as one wants;

    length of longest critical path

Three Recent Theses

- *August 2022, Efficient and Scalable Parallel Functional Programming Through Disentanglement*, by Sam Westrick, advised by Umut Acar.
- June 2022, Deductive Verification for Ordinary Differential Equations: Safety, Liveness, and Stability, by Yong Kiam Tan, advised by André Platzer.
- October 2021, First Steps in Synthetic Tait Computability: The Objective Metatheory of Cubical Type Theory, by Jonathan Sterling, advised by Robert Harper.

Defining ML (Effect-Free Fragment)

- Types t
- Expressions e
- Values v (subset of expressions)

Examples:

$$(3+4)*2$$

$$\stackrel{1}{\Longrightarrow} \; 7 * 2$$

$$\stackrel{1}{\Longrightarrow} \; 14$$

$$(3+4)*(2+1)$$

$$\stackrel{3}{\Longrightarrow} \; 21$$

"the " ^ "walrus"

$\Longrightarrow$ "the walrus"

The expression

"the " ^ "walrus"

reduces to the value

"the walrus".

It has type string.

"the walrus" + 1

$\Longrightarrow$??

The expression

"the walrus" + 1

does not have a type and it does not reduce to a value.

Types

A ***type*** is a **prediction** about the kind of value an expression must have if it winds up reducing to a value.

(SML makes this prediction before evaluating the expression.
Evaluation may ultimately produce a value of that type,
but could alternatively raise an exception or loop forever.)

An expression is ***well-typed*** if it has a type,
and ***ill-typed*** otherwise.

(The phrase '***to type-check e***' means to decide whether ***e*** is well-typed.
The phrase '***e type-checks***' means ***e*** is well-typed.)

Important: SML never evaluates an ill-typed expression.

Given an expression **e**:

First,
SML determines whether **e** is well-typed.

If expression **e** is well-typed,
then SML evaluates expression **e**;
otherwise, SML reports a type error.

Expressions

Every well-formed ML expression `e`

- has a type `t`, written as `e : t`
- may have a value `v`, written as $\texttt{e} \hookrightarrow \texttt{v}$.
- may have an effect (not for our effect-free fragment)

Example: (3+4)*2 : int

(3+4)*2 $\hookrightarrow$ 14

Type `int`

Values $\ldots, \texttt{~1}, \texttt{0}, \texttt{1}, \ldots,$

that is, every integer n.

Expressions e_1 `+` e_2, e_1 `-` e_2, e_1 `*` e_2,

e_1 `div` e_2, e_1 `mod` e_2, *etc.*

Example: ~4 * 3

Typing Rules

- n : `int`
- e_1 + e_2 : `int`

  if e_1 : `int` and e_2 : `int`

similar for other operations.

Example:

$(3+4)*2$: int

Why?

$3+4$: int and 2 : int

Why?

3 : int and 4 : int

Evaluation Rules

- $e_1 + e_2 \xrightarrow{1} e_1' + e_2$ if $e_1 \xrightarrow{1} e_1'$
- $n_1 + e_2 \xrightarrow{1} n_1 + e_2'$ if $e_2 \xrightarrow{1} e_2'$
- $n_1 + n_2 \xrightarrow{1} n$,
  with n the sum of the integer values n_1 and n_2.

Example of a well-typed expression with no value

5 div 0 : int

5 div 0 : int

because 5 : int
& 0 : int
and because div expects two ints and returns an int.

However, 5 div 0
does not reduce to a value.

Notation Recap

$e : t$ "e has type t"

$e \Rightarrow e'$ "e reduces to e'"

$e \hookrightarrow v$ "e evaluates to v"

Extensional Equivalence

$$\cong$$

An equivalence relation on expressions
(of the same type).

Extensional Equivalence

- Expressions are *extensionally equivalent if they have the same type and one of the following is true:*

  both expressions reduce to the same value,

  or both expressions raise the same exception,

  or both expressions loop forever.

- Functions are *extensionally equivalent* if they map equivalent arguments to equivalent results.

- In proofs, we use $\cong$ as shorthand for "is equivalent to".

- Examples: 21 + 21 $\cong$ 42 $\cong$ 6 $*$ 7

  [2, 7, 6] $\cong$ [1+1, 2+5, 3+3]

  (fn x => x + x) $\cong$ (fn y => 2 $*$ y)

- Functional programs are *referentially transparent*, meaning:
  - The *value* of an expression depends only on the *values* of its sub-expressions.
  - The *type* of an expression depends only on the *types* of its sub-expressions.

Need a slightly more general definition to include function values:

- Expressions are *extensionally equivalent* *if they have the same type and one of the following is true:*

  both expressions reduce to equivalent values,

  or both expressions raise equivalent exceptions,

  or both expressions loop forever.

- Functions are *extensionally equivalent* if they map equivalent arguments to equivalent results.

- In proofs, we use $\cong$ as shorthand for "is equivalent to".

- Examples: 21 + 21 $\cong$ 42 $\cong$ 6 $*$ 7

  [2, 7, 6] $\cong$ [1+1, 2+5, 3+3]

  (fn x => x + x) $\cong$ (fn y => 2 $*$ y)

- Functional programs are *referentially transparent*, meaning:
  - The *value* of an expression depends only on the *values* of its sub-expressions.
  - The *type* of an expression depends only on the *types* of its sub-expressions.

Types in ML

Base types:

int, real, bool, char, string

Constructed types:

product types
function types
user-defined types

Types $t_1 * t_2$ for any type t_1 and t_2.

Values (v_1, v_2) for values v_1 and v_2.

Expressions (e_1, e_2), $\underbrace{\#1\ e, \quad \#2\ e}$

DO NOT USE!*

Examples: (3+4, true)

(1.0, ~15.6)

(8, 5, false, ~2)

*You will learn how to extract components using pattern matching

Typing Rules

- $(e_1,\ e_2)\ :\ t_1\ *\ t_2$

  if $e_1 : t_1$

  and $e_2 : t_2$

Example: (3+4 , true) : int * bool

Evaluation Rules

- $(e_1,\ e_2) \stackrel{1}{\Longrightarrow} (e_1',\ e_2) \quad \text{if } e_1 \stackrel{1}{\Longrightarrow} e_1'$

- $(v_1,\ e_2) \stackrel{1}{\Longrightarrow} (v_1,\ e_2') \quad \text{if } e_2 \stackrel{1}{\Longrightarrow} e_2'$

What are the type & value of ...

(3 * 4, 1.1 + 7.2, true)

Type reasoning

$3 * 4$: int

$1.1 + 7.2$: real

true : bool

So $(3 * 4,\ 1.1 + 7.2,\ \text{true})$

: int * real * bool

Evaluation

$$(3 * 4,\ 1.1 + 7.2,\ \text{true})$$
$$\Longrightarrow (12,\ 1.1 + 7.2,\ \text{true})$$
$$\Longrightarrow (12,\ 8.3,\ \text{true})$$

That is a value, so

$$(3 * 4,\ 1.1 + 7.2,\ \text{true})$$
$$\hookrightarrow (12,\ 8.3,\ \text{true})$$

What are the type & value of...

$$(3 * 4,\ 1.1 + 7.2,\ true)$$

$$(3*4,\ 1.1+7.2,\ true) : int * real * bool$$

$$(3*4,\ 1.1+7.2,\ true) \hookrightarrow (12,\ 8.3,\ true)$$

What are the type & value of ...

(5 div 0, 2+1)

(5 div 0, 2+1) : int * int

(5 div 0, 2+1) does <u>not</u> reduce to a value, because evaluation of 5 div 0 raises an exception.

What are the type & value of...

(8 + "miles", false)

This expression is ill-typed, i.e., it has no type, because the subexpression 8 + "miles" is ill-typed.

SML does not evaluate ill-typed expressions, so the expression has no value.

What are the type & value of...

`(2, (true, "a"), 3.1)`

What are the type & value of ...

(2, (true, "a"), 3.1)

This expression has type int * (bool * string) * real, which is **different** from int * bool * string * real.

Contrast:

(2, (true, "a"), 3.1) : int * (bool * string) * real

vs. (2, true, "a", 3.1) : int * bool * string * real.

What are the type & value of...

(2, (true, "a"), 3.1)

(2, (true, "a"), 3.1) : int * (bool * string) * real

(2, (true, "a"), 3.1) $\hookrightarrow$ (2, (true, "a"), 3.1)

Functions

In math, one talks about a function f mapping between spaces X and Y,

$$f : X \rightarrow Y$$

In SML, we will do the same, with X and Y being types.

Issue: Computationally, a function may not always return a value. That complicates checking equivalence.

Def: A function f is ***total*** if f reduces to a value* *and* f(x) reduces to a value for all values x in X.

* (With one unusual exception, this first condition is implied by the second. We write it for emphasis, since f could be a general expression of type $X \rightarrow Y$.)

Functions

In math, one talks about a function f
mapping between spaces X and Y,

$$f : X \to Y$$

In SML, we will do the same, with X and Y being types.

Issue: Computationally, a function may not always return a value. That complicates checking equivalence.

Def: A function f is ***total*** if f reduces to a value *and* f(x) reduces to a value for all values x in X.

(Totality is a key difference between math and computation.)

Sample Function Code

```
(* square : int -> int
   REQUIRES: true
   ENSURES:  square(x) evaluates to x * x
*)

fun square (x:int) : int = x * x



(* Testcases: *)

val 0 = square 0
val 49 = square 7
val 81 = square (~9)
```

Sample Function Code

```
(* square : int -> int
   REQUIRES: true
   ENSURES:  square(x) evaluates to x * x
*)
```

function type

```
fun square (x:int) : int = x * x
```

keyword | function name | argument name & type | result type | body of function

```
(* Testcases: *)

val 0 = square 0
val 49 = square 7
val 81 = square (~9)
```

Five-Step Methodology

1

```
(* square : int -> int
```

function type

2

```
   REQUIRES: true
```

3

```
   ENSURES:  square(x) evaluates to x * x
*)
```

4

```
fun square (x:int) : int = x * x
```

keyword function name argument name & type result type body of function

5

```
(* Testcases: *)

val 0 = square 0
val 49 = square 7
val 81 = square (~9)
```

Six-Step Methodology

1

```
(* square : int -> int
```
function type

2

```
   REQUIRES: true
```

3

```
   ENSURES:  square(x) evaluates to x * x
*)
```

4

```
fun square (x:int) : int = x * x
```

keyword | function name | argument name & type | result type | body of function

5

```
(* Testcases: *)

val 0 = square 0
val 49 = square 7
val 81 = square (~9)
```

6

Proof!

Declarations

Environments

Scope

Declaration

```
val pi : real = 3.14
```

keyword identifier type value

Introduces binding of pi to 3.14
(sometimes written [3.14/pi])

Lexically statically scoped.

```
val x : int = 8-5        [3/x]
val y : int = x+1        [4/y]
val x : int = 10         [10/x]
val z : int = x+1        [11/z]
```

second binding of x
<u>shadows</u> first binding.

First binding has been <u>shadowed</u>.

Local Declarations

let ... in ... end

```
let
    val m : int = 3
    val n : int = m*m
in
    m+n
end
```

This is an expression.

What type does it have? int

What value? 12

Local Declarations

```
val k : int = 4
```

```
let
   val k : real = 3.0
in
   k * k
end
```

$\hookrightarrow$ 9.0 : real

Type?

Value?

k $\leftarrow$ Type?

Value?

$\hookrightarrow$ 4 : int

Concrete Type Def

```
type float = real

type point = float*float

val p:point = (1.0,2.6)
```

Closures

Function declarations also create value bindings:

```
fun square (x:int) : int = x * x
```

binds the identifier **`square`** to a closure.

The closure consists of two parts:

- A lambda expression (code):

```
fn (x : int) => x * x
```

keyword argument name & type body of function

- An environment (all prior bindings).

Closures

Function declarations also create value bindings:

```
fun square (x:int) : int = x * x
```

binds the identifier **`square`** to a closure.

The closure consists of two parts:

- A lambda expression (code):

```
fn (x : int) => x * x
```

keyword

argument name & type

body of function

CAUTION: Do NOT write return type.

- An environment (all prior bindings).

Closures

Function declarations also create value bindings:

```
fun square (x:int) : int = x * x
```

binds the identifier `square` to a closure.

NOTE

The closure consists of two parts:

- A lambda expression (code):

```
fn (x : int) => x * x
```

keyword

argument name & type

body of function

CAUTION: Do NOT write return type.

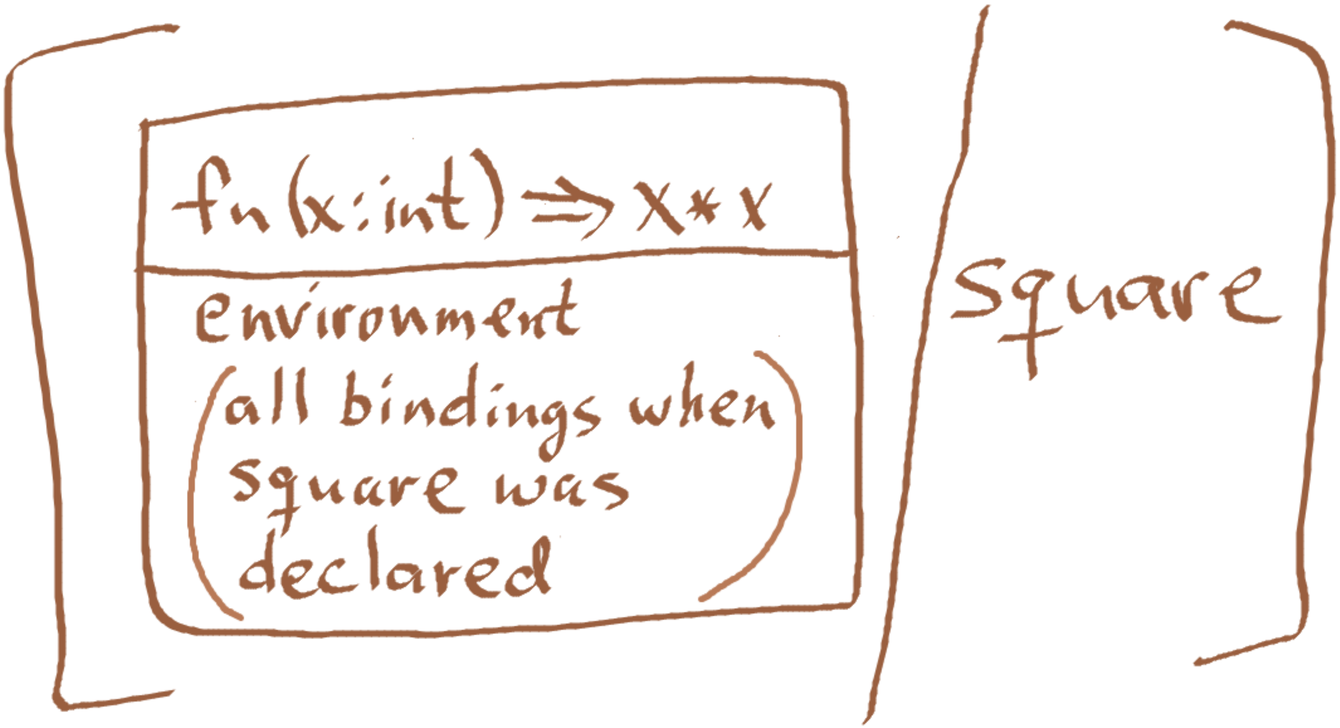
- An environment (all prior bindings).

Closures

Function declarations also create value bindings:

```
fun square (x:int) : int = x * x
```

binds the identifier **`square`** to a closure:



Course Tasks

• Assignments	45%
• Labs	10%
• Midterm 1	10%
• Midterm 2	15%
• Final	20%

Roughly one assignment per week, one lab per week.

Collaboration

Be sure to read the
course and university webpages
regarding academic integrity.

TO DO **TONIGHT**

Go to 150’s **Canvas**.

Select **Assignments**.

Do the **Setup Lab**.

(Important preparation **before** Wednesday’s lab.)

(If you have questions, ask on 150’s Piazza.)

That is all.

Have a good lab tomorrow.

See you Thursday.