15-150

Principles of Functional Programming

Lecture 15

March 13, 2025

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Regular Expressions using Combinators & Staging

Recall from last time:

```
datatype regexp =
             Char of char
           | Zero
           | One
           | Plus of regexp * regexp
           | Times of regexp * regexp
           | Star of regexp
```

Recall from last time:

```
(* match : regexp -> char list ->
                     (char list -> bool) -> bool

   REQUIRES: k is total;
             perhaps also: r is in standard form.

   ENSURES: (match r cs k) returns true if
               cs can be split as cs≅p@s, with
               p representing a string in L(r)
               and k(s) evaluating to true;
          (match r cs k) returns false, otherwise.
*)
```

Recall from last time:

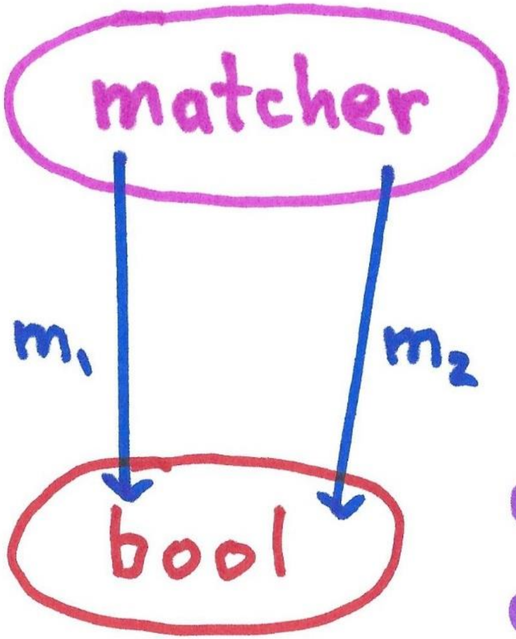
```
(* accept : regexp -> string -> bool
   REQUIRES: perhaps: r is in standard form.
   ENSURES: (accept r s) returns true if s ∈ L(r);
            (accept r s) returns false, otherwise.
*)
```

Implementation

```
  fun match (Char a) cs k =
       (case cs of
           [] => false
         | c::cs' => (a=c) andalso (k cs'))
    | match Zero _ _ = false
    | match One cs k =  k cs
    | match (Plus(r1,r2)) cs k =
       (match r1 cs k) orelse  (match r2 cs k)
    | match (Times(r1,r2)) cs k =
       match r1 cs  (fn cs' => match r2 cs' k)
    | match (Star r) cs k =
        k cs   orelse
        match r cs  (fn cs' => match (Star r) cs' k)

fun accept r s = match r (String.explode s) List.null
```

Today, we will re-implement the regular expression matcher using combinators. Doing so will disentangle the regular expression semantics from I/O (strings and continuations) by providing some staging.



ORELSE
THEN
REPEAT

orelse
andalso

m_i is a matcher for a particular regular expression r_i.

m_1 ORELSE m_2 is a matcher for $r_1 + r_2$.

$$\text{match} : \text{regexp} \rightarrow \overset{\text{conceptual}}{\underset{\text{split}}{\Big\{}} \underbrace{\text{char list} \rightarrow (\text{char list} \rightarrow \text{bool}) \rightarrow \text{bool}}$$

$$\text{regexp} \rightarrow \text{matcher}$$

<u>type</u> matcher = char list → (char list → bool) → bool

Code Outline

Continuation Base Cases

ACCEPT
REJECT

These are matchers

Input Base Case

CHECK_FOR

creates a matcher from a character

Combinators

ORELSE
THEN
REPEAT

combine matchers into new matcher

Overall matcher

match
accept

```
val REJECT : matcher = fn _ ⇒ fn _ ⇒ false
val ACCEPT : matcher = fn cs ⇒ fn k ⇒ k(cs)

val CHECK_FOR (a : char) : matcher =
        fn [] ⇒ REJECT []
         | c::cs ⇒ if a = c
                     then ACCEPT cs
                     else REJECT cs
```

Precedences of predefined infix operators

/ * mod div	7
+ - ^	6
:: @	5
= <> < > <= >=	4
:= o	3

:: & @ are right-associative.

The others are left-associative.

```
infixr 8  ORELSE
infixr 9  THEN

fun (m1 ORELSE m2) cs k  =  m1 cs k  orelse  m2 cs k

fun (m1 THEN m2) cs k  =  m1 cs (fn cs' => m2 cs' k)
```

If regular expressions are in standard form

```
fun REPEAT m cs k =
      let
        fun mstar cs' =
              k cs' orelse m cs' mstar
        in
            mstar cs
        end
```

More generally

```
fun  REPEAT  m  cs  k  =
     let
        fun  mstar  cs'  =
               k cs'  orelse
               m  cs'  (fn  cs" =>
                           properSuffix (cs", cs')
                               andalso
                           mstar  cs")
     in
        mstar  cs
     end
```

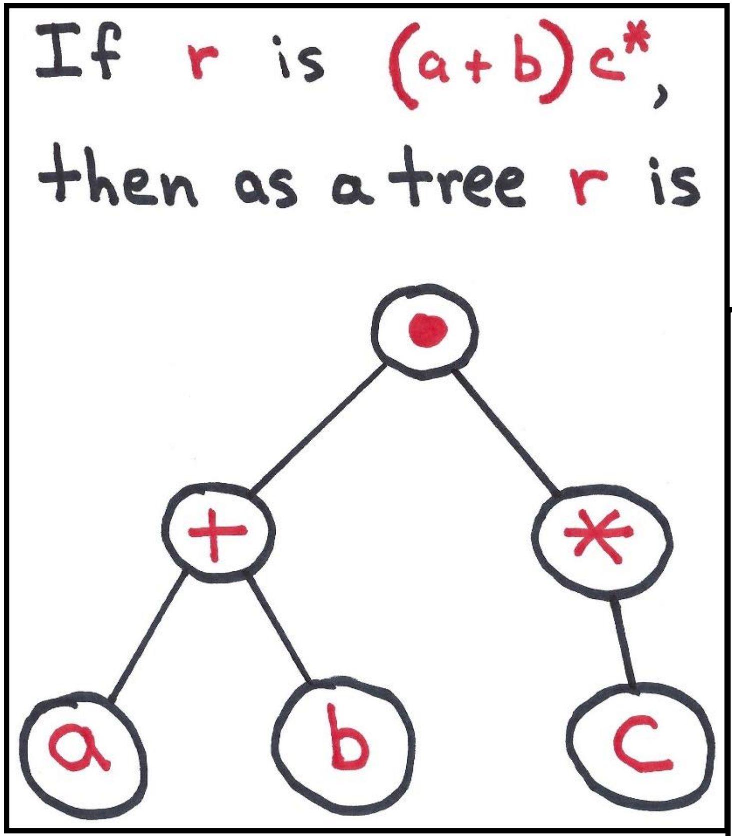
```
fun match (Char(a): regexp): matcher = CHECK_FOR a
  | match Zero              =  REJECT
  | match One               =  ACCEPT
  | match (Plus(r1,r2))     =  match r1  ORELSE  match r2
  | match (Times(r1,r2))    =  match r1  THEN    match r2
  | match (Star(r))         =  REPEAT(match r)
```

```
fun match (Char(a): regexp): matcher = CHECK_FOR a
  | match Zero              = REJECT
  | match One               = ACCEPT
  | match (Plus(r1, r2))    = match r1  ORELSE  match r2
  | match (Times(r1, r2))   = match r1  THEN  match r2
  | match (Star(r))         = REPEAT(match r)
```

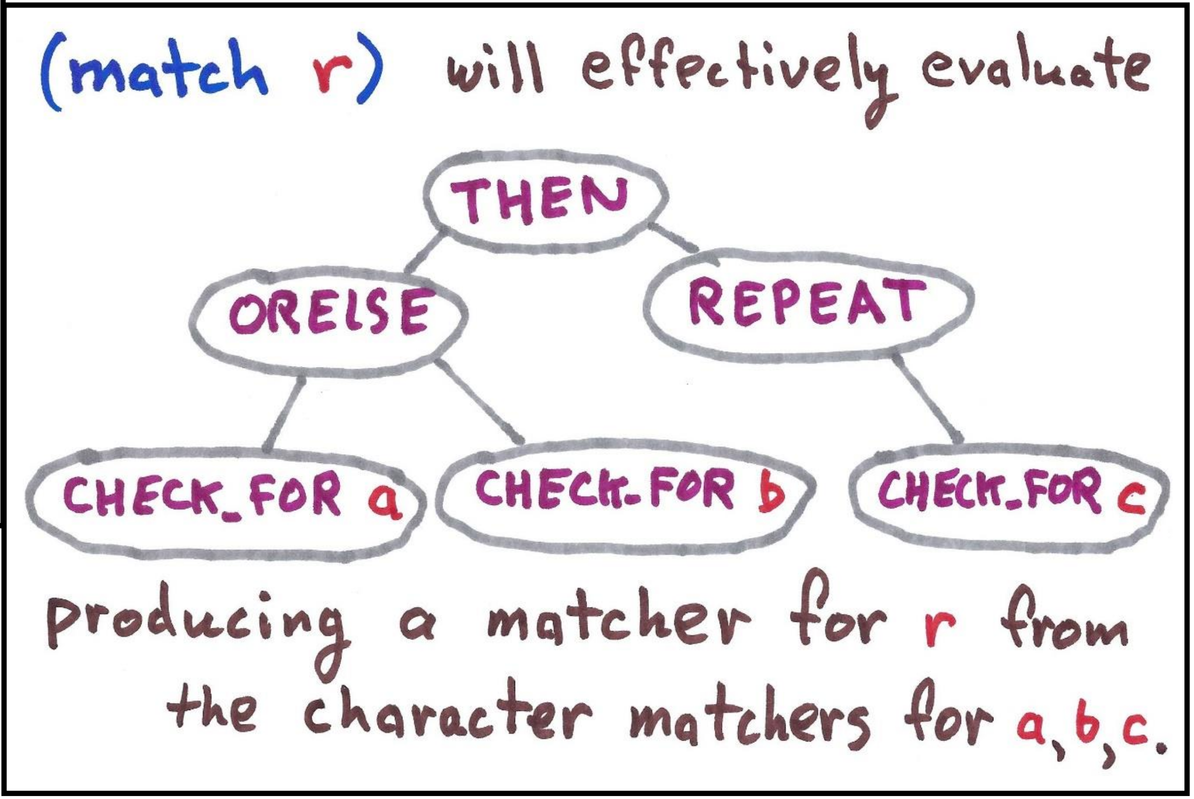
Where is the staging?

In the deconstruction of the regular expression.

(This now occurs before any string or continuation arguments are received.)



If r is $(a+b)c^*$, then as a tree r is



(match r) will effectively evaluate

producing a matcher for r from the character matchers for a, b, c.

This happens *before* any character input or continuations are specified.

We can now stage accept:

```
fun accept (r : regexp) : string → bool =
    let
      val m = match r
    in
      fn s ⇒ m (String.explode s) List.null
    end
```

Previously, evaluation of the expression

accept (Plus(Char #"a", Char #"b"))

would have done very little work, returning nearly instantaneously a function of type

string → bool.

Only after being called on a string would that function have examined the regular expression Plus(⋯).

Now, with staging, accept (Plus(⋯)) builds a matcher for Plus(⋯) right away. That matcher can be re-used for different strings; no need to rebuild it every time.

That is all.

Have a good weekend.

See you Tuesday, when we will start working with Modules.